28[M, X].—PHILIP HARTMAN, *Ordinary differential Equations*, John Wiley & Sons, Inc., New York, 1964, xiv + 612 pp., 24 cm. Price $20.00.

This book treats both the classical theory of differential equations (linear theory, Sturm-Liouville boundary value problems) and the modern nonlinear theory (Poincaré-Bendixson theory, Liapounov's methods, invariant manifolds, stability in the large, applications of fixed-point methods). In addition, there is material found only in the periodical literature (A. J. Schwartz's theory of flows on 2-manifolds, H. Weyl's theory of the generalized Blasius equation of boundary-layer theory).

The writing is clear and meets the high standards one has come to expect from the author. The notes on the literature are especially valuable. To round this all off, this book has one of the best collections of exercises (together with hints) ever seen by this reviewer.

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29[M, X].—PETER HENRICI, *Discrete Variable Methods in Ordinary Differential Equations*, John Wiley & Sons, Inc., New York, 1962, xi + 407 pp., 24 cm. Price $11.50.

How does one solve an ordinary differential equation numerically, and if a method is chosen how accurate will the answer be? These are the questions which this book aims to answer for the method of finite differences. In short, it presents an elaborate analysis of methods that are used on today's computers.

The book is divided into three parts: one-step methods, multi-step methods, and two-point boundary value problems. The last part makes up only about one-seventh of the book, while the first occupies about one-half. The second part gives us, for the first time in a text, a presentation of some of the beautiful results of Dahlquist on multi-step methods.

The mode of presentation in each part is to present a method for solving a differential equation problem, followed by an existence and convergence proof. The author then proceeds to give an elaborate discussion of error propagation and rate of convergence. Thus, in the first part, we find a detailed account of Euler's method (for one equation of first order), which includes not only a constructive existence theorem but also a complete analysis of error propagation from the usual and from a probabilistic point of view. This is expanded to general one-step methods with applications to the methods of Runge, Kutta, Heun, and others. This analysis is subsequently repeated for systems of first- and higher-order equations. For this first half of the book only a knowledge of calculus and a modest knowledge of matrix calculus and probability is called for.

The multi-step methods are introduced by examining the methods of Adams, Bashforth, Moulton, Nyström, and Milne. This is followed by the more general case with a presentation of some of the theorems of Dahlquist. Here we are given necessary and sufficient conditions for convergence of such schemes. Included is a discussion of the properties of stable operators and the startling results of Dahlquist on the maximal order achievable with such operators. Only a minimal knowledge of complex variable theory is required.

The last part discusses Newton's method in n-dimensional space, and this is then applied to a second-order two-point boundary-value problem. In all cases we find very detailed discussions of the discretization errors and other numerical errors.

Each chapter has a generous number of exercises of varying degrees of complexity plus a useful bibliographical summary. Most of the methods are illustrated by examples and flow charts.

Apart from a few minor misprints, which do not mar the excellence of the presentation, we have only the following notes of criticism: The uninitiated reader should be prepared for a large variety of "errors" to be encountered. Apart from the usual discretization and round-off errors, of both the "local" and "genuine" variety, we find induced, adduced, inherent, starting, accumulated (both primary and secondary), and magnified errors. There is a tendency to couple and uncouple words such as stepnumber, which appears uncoupled in the index. The row sum given on p. 371 is evaluated on p. 375. It is not true, in general, that if algorithm (7–21) breaks down, then A is singular. Is there an extra hypothesis to be made in (7–74) that L_2 exists?

The text is otherwise carefully written and is a welcome addition to the growing body of literature on the analysis of finite-difference methods. This work will, I am sure, enlighten those interested in both discrete problems as well as non-discrete (or is it in-discrete) problems.

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30[M, X].—ALEKSANDR SEMENOVICH KRONROD, *Nodes and Weights of Quadrature Formulas*, Consultants Bureau, New York, 1965, vii + 143 pp., 28 cm. Price $12.50.

This book is a translation of a Russian book published in 1964. It contains tables of two types of quadrature formulas: Gauss-Legendre and so-called "improved" quadrature formulas.

The Gauss-Legendre formulas are the well-known approximations of the form

$$\int_0^1 f(x)\,dx \simeq \sum_{i=1}^{N} A_i f(\nu_i) \tag{1}$$

which are exact for all polynomials of degree $\leqq 2N - 1$. The A_i and ν_i in (1) are tabulated for $N = 1(1)40$.

Also tabulated are formulas of the form

$$\int_0^1 f(x)\,dx \simeq \sum_{i=1}^{N} A_i^* f(\nu_i) + \sum_{j=1}^{N+1} B_j f(\mu_j) \tag{2}$$

which are called improved formulas. In (2) the ν_i are the points in (1); the A_i^*, B_j and μ_j are chosen so that (2) is exact for all polynomials of the highest possible degree k; for N even, $k = 3N + 1$, and for N odd, $k = 3N + 2$. The constants in (2) are also given for $N = 1(1)40$. From the tables it is seen that the μ_j separate the ν_i, but no proof is given.